Efficiently Assigning Bogey Teams

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Abstract

Unlike most sports tournaments, the 2025 *Counter-Strike* Blast Bounty tournament lets teams pick their opponents. The tournament organizer allows lower rated teams, called *bounty hunters*, pick a higher rated opponent, with the hopes that higher rated teams are forced to play against their bogey teams, or the teams they are weak against. I first describe the maximization problem of the team and the organizer, and show that the existing mechanism in the Blast Bounty tournament does not maximize the utility of the organizer. I then present a mechanism that can maximize the organizers utility, as well as the conditions any such mechanism that aims to do so has to satisfy.

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1 Introduction

Sports tournaments are designed to be as fair as possible. They should accurately identify the best competitor, while keeping viewers interested. This has resulted in most tournaments resembling each other. Most competitions are either round-robins, single- or double-elimination brackets, swiss-systems, or some combination of these mechanisms. However, there is an interest from viewers for different and new ways to structure tournaments. Proposals for a new Swiss Super League format, for example, were heavily debated and went through multiple revisions before finally going into effect in 2023. Similarly, the recent UEFA Champions League changes to the group stage met widespread disapproval initially. After the 2024 tournament took place with the new format, fans warmed up to the increased diversity of games and overall excitement.

In 2024, the e-sports tournament organizer Blast Premier asked itself how it could make its *Counter-Strike* tournaments more engaging and different from other tournament organizers. *Counter-Strike* tournaments almost always consist of a group stage, followed by a single- or double- elimination playoff. These tournaments result in high-level engaging matches in elimination stage, after most of the less performing teams have been eliminated. The group stages that select the top teams for the playoffs, however, hasve not been designed to excite the spectators. With the hopes of making the group stage, as well as the playoffs, more enjoyable to watch, Blast Premier introduced the Blast Bounty tournament.

Blast Bounty[2] invites the 32 highest ranked teams in the Valve Global rankings[1] and seeds the teams according to their ranking. The teams are divided into two groups of 16. The 16 highest ranked teams get assigned a bounty, where the highest ranked team gets the largest bounty, and the rank 16 team the smallest. The remaining 16 teams are bounty hunters and do not start with a bounty. Before round one, the bounty hunters get to choose which team they want to face. This is done with a serial dictatorship where the order is determined by rank, the highest ranked picking its opponent first. The matches are played after all bounty hunters have picked their opponent.

The tournament is single elimination; teams that lose in round one go home empty-handed. Winning a match grants the winner half of its opponent's bounty in cash, and adds the other half to the winning teams bounty. A team that beats a higher seeded team inherits its seed. This process repeats one more time: Seeds 9-16 are assigned to the bounty hunter group and may choose their matchup. Since winning a games grants the winner half its opponent's bounty, every team in round 2 will have a bounty. The total sum of bounties available in round two is smaller or equal to the sum in round one, depending

[1] Blast Bounty only invites the 28 highest ranking teams, the other 4 teams are selected by the organizer

on how many bounty hunters prevailed in round one. The remaining 8 teams play a more traditional single-elimination bracket to determine the winner. The bounty system, however, stays in place until the final. For simplicity, I will be restricting my analysis to round one.

2 Definitions

The tournament has n teams split into 2 equally large groups. Teams are assigned into groups based on their world ranking, with the $n/2$ highest ranked teams being assigned to the *top teams* group, and the lower ranked teams assigned to the *bounty hunters* group. Teams are also assigned seeds according to their world ranking, with *the top teams* being seeds 1-16. The world ranking is an ordinal ranking of the team's skill. For all teams, the win probability playing against seed 1 is smaller or equal than the win probability playing against seed 2.

The top teams group is $\mathcal{T} = \{T_1, T_2, \ldots, T_{n/2}\}$ and the bounty hunters group is $\mathcal{B} = \{B_1, B_2, \ldots, B_{n/2}\}$. Top team T_j starts the tournament with a bounty Y_j and bounty hunters start without a bounty. Bounty hunters are granted $1/2Y_J$ in cash if they beat T_j, and $1/2Y_j$ is added to their bounty.

Teams have ratings (Elo, Glicko). The *expected win probability* $\hat{p}_{ij}$ is the probability that a team i wins against opponent j based on the ratings of both teams. The true, unobservable, win probability of team i versus opponent j is π_{ij}. A team i is considered a *bogey team* of the team j if $\hat{p}_{ij} < \pi_{ij}$. In contrast, team j is a *favorable team/matchup* of team i in this case. π_{ij} is known to the team i and j, but not to the organizer. The percentage difference $\frac{\pi_{ij}-\hat{p}_{ij}}{\hat{p}_{ij}}$ is how *strong* a bogey team is, or how *favorable* the matchup g_{ij} is.

g_{ij} is a binary variable that is 1 if team B_i is matched against T_j, and 0 otherwise. The goal of the organizer is to match teams to their bogey teams, specifically each of the top teams with their strongest bogey team, if possible. The utility of the organizer U_o is the squared sum of the strength of bounty teams.

$$\sum_{i=1}^{n/2}\sum_{j=1}^{n/2} g_{ij} \cdot \left(\frac{\pi_{ij} - \hat{p}_{ij}}{\hat{p}_{ij}}\right) \cdot \left|\left(\frac{\pi_{ij} - \hat{p}_{ij}}{\hat{p}_{ij}}\right)\right| \tag{1}$$

The optimal allocation for the organizer is the allocation where the sum of the squared strength of the bogey matchups is maximal:

$$max \sum_{i=1}^{n/2}\sum_{j=1}^{n/2} g_{ij} \cdot \left(\frac{\pi_{ij} - \hat{p}_{ij}}{\hat{p}_{ij}}\right) \cdot \left|\left(\frac{\pi_{ij} - \hat{p}_{ij}}{\hat{p}_{ij}}\right)\right| \tag{2}$$

Since a top team can be a bogey team of a bounty hunter, the difference may be negative. The organizer wants to match top teams to their bogey teams and not a favorable team, which is why the difference is multiplied with its absolute value, and not squared.

Teams are risk neutral and simply seek to maximize their profit. The utility of team B_i is:

$$U_i = \pi_{ij} \cdot Y_j \tag{3}$$

3 The existing mechanism

The 2025 Blast Bounty tournament allocated bounty hunters to top teams using serial dictatorship. The bounty hunters, in order of their ranking/seed, pick their opponents, with the highest seeded bounty hunter, seed 17, picking first. The bounties are determined based on the seeding of the teams. Seeds 1-4, 5-8, 9-12, and 13-16 are given \$60‘000, \$30‘000, \$15‘000, and \$12‘000 bounties respectively.

This mechanism fails to produce the optimal allocation for two reasons. First, bounty hunters will pick the opponent T_j where $\pi_{ij} \cdot Y_j$ is maximal, which means that the estimated win probability $\hat{p}_{ij}$ is not relevant for the team's decision-making. Second, even if bounty hunters always picked a favorable matchup, it would fail to result in an optimal allocation (consistently), as serial dictatorship is a greedy mechanism. If B_1 and B_2 are bogey teams of T_3, but B_2 is a stronger bogey team of T_3, B_1, who get to pick first, would pick T_3.

The goal of the organizer is to create a mechanism where teams maximizing their utility also maximizes the utility of the organizer. The utility of the organizer U_o of game g_{ij} is a function of $\hat{p}_{ij}$ and π_{ij}, $U_o(g_{ij}) = U(\hat{p}_{ij}, \pi_{ij})$. Teams simply seek to maximize their revenue, and so their utility U_i of game g_{ij} is a function of π_{ij} and Y_j, $U_i(g_{ij}) = U(Y_j, \pi_{ij})$. Since $\hat{p}_{ij}$ has no impact on the decisions of the teams, and Y_i has no impact on the utility of the organizer, the existing mechanism does not fulfill the organizers goal. Thus, the organizer has to design the mechanism in a way that teams care about $\hat{p}_{ij}$. This can be achieved by making Y_i dependent on $\hat{p}_{ij}$.

Let g_{ij} be the matchup between teams B_i and T_j, with the set of attributes $A = \{a_1, a_2, \ldots, a_n\}$. The average age of players, the variance in height of players, the expected duration of the game are examples of attributes a_{ij}. Here, $\hat{p}_{ij}$ and π_{ij} are the only attributes of game g_{ij} in A that are of interest. U_o and U_i are the utilities of the organizer and team B_i respectively.

$$sign\left(\frac{\partial U_o}{\partial a_i}\right) = sign\left(\frac{\partial U_{B_i}}{\partial a_i}\right), \forall a_i \in A \text{ if } \frac{\partial U_o}{\partial a_i} \neq 0 \tag{4}$$

or[2]:

$$\frac{\partial U_o}{\partial a_i} \cdot \frac{\partial U_{B_i}}{\partial a_i} > 0, \forall a_i \in A \text{ if } \frac{\partial U_o}{\partial a_i} \neq 0 \tag{5}$$

Any mechanism that does not fulfill this condition will not maximize the utility of the organizers.

4 The Bounty Y_j

I have shown that the organizer must design a mechanism where $\frac{\partial U_{B_i}(g_{ij})}{\partial \hat{p_{ij}}} \neq 0$. In other words, $\hat{p}_{ij}$ needs to affect a team's utility, and consequently its decision-making. The utility of team B_i is $U_{B_i}(g_{ij}) = \pi_{ij} \cdot Y_j$, with the organizer only able to influence the bounty Y_j. In order to maximize its utility, the organizer must define Y_j so that $\frac{\partial Y_j}{\partial \hat{p_{ij}}} \neq 0$.

The utility of the organizer of game g_{ij} is decreasing in $\hat{p}_{ij}$; Ceteris paribus, a higher expected win probability $\hat{p}_{ij}$ means the difference between π_{ij} and $\hat{p}_{ij}$ is smaller. The utility of team B_i of game g_{ij} is increasing in Y_j, so Y_j needs to be decreasing in $\hat{p}_{ij}$.

$$\frac{\partial U_o(g_{ij})}{\partial \hat{p}_{ij}} < 0 \tag{6}$$

$$\frac{\partial U_{B_i}(g_{ij})}{\partial \hat{Y}_j} > 0 \tag{7}$$

$$\Rightarrow \frac{\partial Y_j}{\partial \hat{p_{ij}}} < 0 \tag{8}$$

This has its own problems for the organizer: $\hat{p}_{ij}$ is an attribute of a game g_{ij}. Y_j so far has been the bounty on team T_j. If Y_j is a function of $\hat{p}_{ij}$, then it cannot be an attribute of a team, but an attribute of a matchup g_{ij}. In other words, no mechanism where the bounty Y is an attribute of a team can result in an optimal allocation for the organizer. The bounty Y_j of team T_j has to be different for team B_1 than for team B_2. From now on, I will be referring to the bounty on T_j for B_i as Y_{ij}.

[2] Equation 6 is more elegant, while 5 is more intuitive. They are mathematically identical.

5 A better mechanism

The remaining challenge for the organizer is how exactly to set bounties Y_{ij} based on $\hat{p}_{ij}$:

The organizer assigns a budget to each bounty hunter in $\mathcal{B}$. Then, bounties Y_{ij} are assigned so that team B_i would be indifferent between any matchup g_{ij} in $\mathcal{T}$ if $\hat{p_{ij}} = \pi_{ij}$. That is to say, if there were no bogey teams, then every single allocation would be pareto-efficient and envy-free.

$$\hat{p}_{ij} \cdot Y_{ij} = c_i, \forall T_j \in \mathcal{T} \tag{9}$$

And:

$$\sum_{j}^{n/2} Y_{ij} = M, \forall B_i \in \mathcal{B} \tag{10}$$

So if B_i is twice as likely to beat T_1 than T_2, $\hat{p}_{i,1} = 2\hat{p}_{i,2}$, then $2Y_{i,1} = Y_{i,2}$.

$$\frac{\hat{p}_{i,1}}{\hat{p}_{i,2}} = \frac{Y_{i,2}}{Y_{i,1}} \tag{11}$$

Each bounty hunter B_i is given the list of opponents and their corresponding bounties Y_{ij}. Now, the organizer hosts a modified *Demange-Gale-Sotomayor* (DGS)[3] auction, which is a has strong similarities to the second-price auction and the English auction.

The DGS mechanism The *approximate auction mechanism* by Demange, Gale, and Sotomayor starts with the auctioneer announcing an initial sales price. Any bidder can bid for an item, which obligates the bidder to buy the item for the current price if no other bid is placed on it. The bidder is considered tentatively assigned to that item (similar to the DA algorithm). If a bidder bids on an item that is already assigned to another bidder, the price is increased by a fixed amount δ and the new bidder is assigned to the item. Once every item is assigned to a bidder, the auction terminates. The bidder buy their assigned items for the current price.

For the tournament, there are a few changes: The bids start at 0 and represent how high of a *percentage* α_{ij} of the bounty Y_{ij} team B_i is willing to give up. α_j is the price of an opponent T_j. If a team B_i bids on team T_i, then the price α_j is increased by δ. The final bounty Y_j of T_j for the team B_i that gets assigned the matchup will therefore be $(100 - \alpha_j)/100 \cdot Y_{ij}$. The auction is analogue to an English auction, as the bidders only need to bid until no other bidder bids on the same opponent, and therefore teams are incentivized to bid their true value. The modified DSG-auction is manipulable by groups of bidders, however.[3]

The organizer has multiple ways to run the auction, none of which change the outcome, but may make the bidding process more intuitive for the bounty hunters. Teams can submit more conventional bids, where the bid b_{ij} is how much a team B_i is willing to pay for a certain matchup g_{ij}. In this case the organizer has to convert the bid b_{ij} to the percentage difference α_{ij}. For example, if team B_i has a bounty Y_{ij} on team T_{ij} and a bid b_{ij}, the percentage bid α_{ij} is $100 \cdot (b_{ij}/Y_{ij})$. It might be more intuitive for teams to bid a certain monetary value, rather than a percentage of the bounty they are willing to give up. However, this means the auction has to be a sealed bid auction, since it's not the bids b_{ij} that are getting compared, but the percentage bids α_{ij}.

As already stated, the auction can be held both as a sealed bid auction, or an open outcry auction. Both of these have the same outcome, in theory. In practice, depending on how large δ is set, the auction will either take a very long time, if δ is small, or not necessarily produce the optimal outcome, if δ is large.

In the open outcry auction, teams have the list of bounties in front of them. More accurately, the teams get a constantly updating list of $(100-\alpha_j)/100 \cdot Y_{ij}$, where α_j is the current price of team T_j. Teams pick the team T_j they want to face, and if the team T_j was already assigned, α_j gets increased by δ. This continues until every team in $\mathcal{T}$ has been assigned assigned a team in $\mathcal{B}$. In the sealed bid version, teams submit bids, and the auction is run by the organizer. This lets the organizer make δ much smaller, as the algorithm can be run by a computer.

[3] An example of this is shown on page 9

6 From start to finish

Consider a tournament with 6 teams. 3 in $\mathcal{B}$ and 3 in $\mathcal{T}$. Table 1 shows the two attributes $\hat{p}_{ij}$ and π_{ij} of every possible game g_{ij}. This table represents the information known by the team, although it is not necessary that the teams know $\hat{p}_{ij}$ for the mechanism to function. Note that π_{ij} is only know to B_i, and not other teams in $\mathcal{B}$.

Step 1: The organizer starts by deciding on a budget M for the bounties available to each team. In this example, M is 100. The organizer now sets the bounty by setting the bounties so that $\hat{p}_{ij} \cdot Y_{ij} = c_i, \forall T_j \in \mathcal{T}$. For B_i this is solved by solving the following system of linear equations:

$$0.3 \cdot Y_{1,1} = 0.4 \cdot Y_{1,2} = 0.5 \cdot Y_{1,3} \tag{12}$$

$$Y_{1,1} + Y_{1,2} + Y_{1,3} = 100 \tag{13}$$

g_{ij}	B_1		B_2		B_3	
a_{ij}	$\hat{p}_{1,j}$	$Y_{1,j}$	$\hat{p}_{2,j}$	$Y_{2,j}$	$\hat{p}_{3,j}$	$Y_{3,j}$
T_1	0.3	42.5	0.2	46.1	0.1	57.1
T_2	0.4	32	0.3	30.8	0.2	28.6
T_3	0.5	25.5	0.4	23.1	0.4	14.3
c_i	12.8		9.2		5.7	

Table 1: $\hat{p}_{ij}$ and Y_{ij} for matchup g_{ij}

Step 2: The result is shown in table 1. The organizer gives the list of $Y_{1,j}$ to B_1, $Y_{2,j}$ to B_2, and $Y_{3,j}$ to B_3. The teams, having received their list of bounties, now calculate their utility of each matchup g_{ij}. Table 2 shows the the expected win probability and true win probability of team B_i for each matchup, as well as which teams are favorable matchups for each team in $\mathcal{B}$. Here, B_1 is a bogey team of T_2, B_2 is a bogey team of T_1 and T_2, and B_3 is a bogey team of T_3.

Step 3: Table 3 shows the utility U_i of matchup g_{ij} for team B_i. Since teams are risk neutral and profit maximizing, the utility is simply the expected value $\pi_{ij} \cdot Y_{ij}$. The teams submit their bids α_{ij}, which represents how high of a percentage of bounty Y_{ij} they are willing to give up to face team T_j. α_{ij} is given by the equation:

$$\alpha_{ij} = \frac{U_{ij} - Y_{ij}}{U_{ij}} \cdot 100 \tag{14}$$

g_{ij}	B_1		B_2		B_3	
a_{ij}	$\hat{p}_{1,j}$	$\pi_{1,j}$	$\hat{p}_{2,j}$	$\pi_{2,j}$	$\hat{p}_{3,j}$	$\pi_{3,j}$
T_1	0.3	0.3	0.2	0.3	0.1	0.1
T_2	0.4	0.5	0.3	0.4	0.2	0.2
T_3	0.5	0.5	0.4	0.4	0.4	0.5
Favorable Matchups	T_2		T_1, T_2		T_3	

Table 2: $\hat{p}_{ij}$ and π_{ij} for matchup g_{ij}

g_{ij}	B_1			B_2			B_3		
a_{ij}	$Y_{1,j}$	$\pi_{1,j}$	U_1	$Y_{2,j}$	$\pi_{2,j}$	U_2	$Y_{3,j}$	$\pi_{3,j}$	U_3
T_1	42.5	0.3	12.8	46.1	0.3	13.8	57.1	0.1	5.7
T_2	32	0.5	16	30.8	0.4	12.3	28.6	0.2	5.7
T_3	25.5	0.5	12.8	23.1	0.4	9.2	14.3	0.5	7.2
c_i	12.8			9.2			5.7		

Table 3: $\hat{p}_{ij}$ and π_{ij} for matchup g_{ij}

Bids	$\alpha_{1,j}$	$\alpha_{2,j}$	$\alpha_{3,j}$
T_1	0%	33%	0%
T_2	20%	25%	0%
T_3	0%	0%	20%

Table 4: Bids α

Step 4: The organizer uses Table 4 to run the DSG-auction. In this example, the auction concludes almost instantly. B_1 picks T_2, B_2 picks T_1, and B_3 picks T_3. Since every team B_i picked an unassigned opponent, α_j is 0 for all teams T_j. Considering the fact that the DSG-auction is a generalization of the second-price auction, this makes sense: The winning bidder pays the second highest bid (excluding those bids that were larger than the winning bid), which happens to be 0% here.

Bids	$\alpha_{1,j}$	$\alpha_{2,j}$	$\alpha_{3,j}$
T_1	0%	33%	40%
T_2	20%	25%	0%
T_3	0%	0%	20%

Table 5: B_3 is a bogey team of T_1

Imagine now that B_3 is also a bogey team of T_1, and the steps 1-3 yielded the bids shown in table 5. The auction would run in the following manner:

1. B_1 picks T_2
2. B_2 picks T_1
3. B_3 picks T_1, α_1 increases by δ
4. B_2 and B_3 go back and forth picking T_1 until $\alpha_1 = 7$.
5. B_1 is willing to pick T_2 as long as $\alpha_2 < 20\%$, and B_3 is willing to pick T_1 as long as $40\% - \alpha_1 > 20\%$
6. B_2 alternates between bidding on T_1 and T_2. Every time B_2 bids on T_1 or T_2, it is outbid by B_3 and B_1 respectively.
7. This goes on until $\alpha_1 = 20\%$, $\alpha_2 = 13\%$, $\alpha_3 = 0\%$
8. Now, α_1 has risen so much that B_3 now prefers T_3. B_3 picks the previously unassigned T_3. Every team in $\mathcal{T}$ is now assigned an opponent and the auction terminates.
9. The allocation is $\{(B_1, T_2), (B_2, T_1), (B_3, T_3)\}$.
10. The final prices are: $\alpha_1 = 20\%$, $\alpha_2 = 13\%$, and $\alpha_3 = 0\%$
11. The final bounties are: $Y_1 = 80\% \cdot Y_{2,1}$, $Y_2 = 87\% \cdot Y_{1,2}$, and $Y_3 = 100\% \cdot Y_{3,3}$[4]

Despite the fact that B_3 is a strong bogey team of T_1, the allocations resulting from the bids in table 4 and table 5 are identical. The bids in table 4 however, lead to higher bounties for B_1 and B_2 and are a pareto improvement over the bounties resulting from bids in table 5. While a team B_i doesn't gain utility from lying, other teams can gain from B_i lying. In this case, B_1 and B_2 can collude and pay B_3 a small amount to not bid on T_1.

[4]You can simulate these auctions here[1]

7 Conclusion

I have shown why the current mechanism of the Blast Bounty tournament fails to achieve the goal set out by the organizer, and provided a mechanism that does. The mechanism differs from conventional auctions due to the seller giving bidders "private prices", and the bids being ratios of those "private prices". Here, the private prices were the bounties Y_{ij}.

In most auctions, the seller cares only about its own profit, or solving assignment problems efficiently. The framework in this paper is useful for analyzing situation where the seller or organizer cares about some other attribute of a matching, like whether teams face their bogey teams.

The idea of this paper goes beyond tournaments. It is generally applicable to situations where the seller has a preference for specific matchings, beyond simple profit maximization. For example, a parent writing their will may not be sure how exactly to allocate their wealth among their children. However, the parent does not wish to auction the inheritance between their children, as the parent still wants to have some sway over the allocation. The parent may prefer giving their boat to their youngest child, as that was the child that spent the most time on it. Deciding unilaterally which child gets what will inevitably lead to envy and discontent, something the parent wishes to avoid. So, like the organizer, the parent can offer the children different prices for the boat, and auction it off to the child who is willing to pay the highest mark-up over the starting price. In that way, if the youngest child wants it much less than an other child, the boat will go to the other child, but if the bids are close, the parent has "tipped the scales" in favor of the youngest.

References

[1] Martin Aumüller. Auction matching algorithm, 2013. Retrieved May 9, 2025.

[2] BLAST.tv. Blast bounty: A dynamic prize pool that will put a target on the world's elite, 2024. Retrieved May 9, 2025.

[3] Gabrielle Demange, David Gale, and Marilda Sotomayor. Multi-item auctions. *Journal of Political Economy*, 94(4):863–872, 1986.